DEPARTMENT OF MATHEMATICS

Ph.D (Mathematics)

ACADEMIC SESSION 2010 onwards

(Updated from March 2023)

J. C. BOSE UNIVERSITY OF SCIENCE AND TECHNOLOGY, YMCA

J.C.BOSE UNIVERSITY OF SCIENCE AND TECHNOLOGY, YMCA FARIDABAD
DEPARTMENT OF MATHEMATICS

SCHEME OF Ph.D MATHEMATICS (Course Work) (2010)

Subject code	Title	L	T	P	Sessional Marks	Final Exam Marks	Total	Credits	Category Code
PHAS-06	Operation Research	4	0	0	40	60	100	4	DCC
PHAS-07	Fixed Point Theory	4	0	0	40	60	100	4	DCC
PHD-100A	Research Methodology	4	0	0	40	60	100	4	DCC

SCHEME OF Ph.D MATHEMATICS (Course Work) (2016)

Subject code	Title	L	T	P	Sessional Marks	Final Exam Marks	Total	Credits	Category Code
PHAS-09	Reliability Theory and its Application	4	0	0	40	60	100	4	DCC
PHD-100A	Research Methodology	4	0	0	40	60	100	4	DCC

SCHEME OF Ph.D MATHEMATICS (Course Work) (2018)

Subject code	Title	L	T	P	Sessional Marks	Final Exam Marks	Total	Credits	Category Code
PHDM-01	Operation Research	4	0	0	25	75	100	4	DCC
PHD-100A	Research Methodology	4	0	0	25	75	100	4	DCC

SCHEME OF Ph.D MATHEMATICS (Course Work) (2019)

Subject code	Title	L	T	P	Sessional Marks	Final Exam Marks	Total	Credits	Category Code
PHDM-01	Operation Research	4	0	0	25	75	100	4	DCC
PHDM-02	Quantum Optics	4	0	0	25	75	100	4	DCC
PHD-100A	Research Methodology	4	0	0	25	75	100	4	DCC

SCHEME OF Ph.D MATHEMATICS (Course Work) (2020)

Subject code	Title	L	T	P	Sessional Marks	Final Exam Marks	Total	Credits	Category Code
PHDM-01	Operation Research	4	0	0	25	75	100	4	DCC
PHDM-02	Quantum Optics	4	0	0	25	75	100	4	DCC
PHDM-03	Advanced Mechanics of Solids	4	0	0	25	75	100	4	DCC
PhD-100A	Research Methodology	4	0	0	25	75	100	4	DCC

SCHEME OF Ph.D MATHEMATICS (Course Work) (2022)

Subject code	Title	L	T	P	Sessional Marks	Final Exam Marks	Total	Credits	Category Code
PHDM-01	Operation Research	4	0	0	25	75	100	4	DCC
PHDM-02	Quantum Optics	4	0	0	25	75	100	4	DCC
PHDM-03	Advanced Mechanics of Solids	4	0	0	25	75	100	4	DCC
PHDM-04	Approximation Theory	4	0	0	25	75	100	4	DCC
PhD-100A	Research Methodology	4	0	0	25	75	100	4	-
CPE-RPE	Research and Publication Ethics	2	0	0	25	75	100	2	-

Syllabus of Operations Research for Ph.D Coursework

Code: PHAS-06

UNIT 1: Linear Programming Problem:

(i) Computational procedure of simplex method
(ii) Alternative optimum solutions: unbounded solutions, non-existing feasible solutions.
(iii) Big M method, Two phase method
(iv) Sensitivity Analysis
(v) Revised Simplex Method
(vi) Duality, Dual Simplex Method

UNIT 2: Inventory Models

(i) Various costs
(ii) Deterministic inventory models
(iii) Single period inventory model with shortest cost
(iv) Stochastic models
(v) Application of inventory models
(vi) Economic lot sizes-price breaks

UNIT 3: Queuing theory

(i) Introduction to Queues
(ii) Basic Elements of Queuing Models
(iii) Queue Disciplines
(iv) Role of Exponential and Poisson Distributions
(v) Markovian Process, Erlang Distribution
(vi) Distribution Of Arrivals, Distribution of Service Times
(vii) Definition of Steady and Transient State, Poisson Queues.

UNIT 4: Integer linear programming

(i) Integer Linear Programming Problems
(ii) Mixed Integer Linear Programming Problems
(iii) Cutting Plane Method
(iv) Branch and Bound Method
(v) 0-1 integer linear programming problem.

UNIT 5: Non linear programming & Quadratic programming

(i) Formulation of non linear programming problems
(ii) General non linear programming
(iii) Canonical form of non linear programming problem
(iv) Kuhn Tucker conditions, non negative constraints
(v) General quadratic programming problem
(vi) Wolfe's modified simplex method
(vii) Beale's method

UNIT 6: Dynamic Programming

(i) Bellman's Principle of optimality of Dynamic Programming
(ii) Multistage decision problem and its solution
(iii) Solution of linear programming problems as a Dynamic Programming Problem

Books

- Hadley, G.,"Linear Programming, and Massachusetts", Addison-Wesley
- Taha, H.A, "Operations Research – An Introduction", Macmillian
- Hiller, F.S., G.J. Lieberman, " Introduction to Operations Research", Holden-Day
- Harvey M. Wagner, "Principles of Operations Rsearch with Applications to Managerial Decisions", Prentice Hall of India Pvt. Ltd.
- Swarup K., "Operations Research", S. Chand

Syllabus for Pre-Ph.D. Programme

Fixed Point Theory

Section-I

Metric spaces and topological spaces, neighbourhoods, limit points, open and closed sets, completeness of metric spaces, Cantor's intersection theorem, contraction mappings, contraction principle and its converse, continuous and uniform continuous functions in metric spaces, compact metric spaces.

Section-II

Normed linear spaces, Banach spaces, fixed points, Lipschitz, non-expansive, contractive, contraction mappings and relation between these mappings, retraction mapping, $\in$ - chain and $\in$ - chainable metric spaces. Brouwers and Schauder's fixed point theorems and applications.

Section-III

Probabilistic metric spaces, Fuzzy Metric Spaces, Intuitionistic fuzzy metric Spaces, Various types of mappings such as commuting mappings, compatible mappings and their variants.

Suggested books:

1. Istratescu, V.I., Fixed Point Theory: An Introduction, Springer.

2. Joshi, M.C. and Bose, R.K., Some Topics in Non-Linear Functional Analysis, John Wiley & Sons (Asia).

3. J. Dugundji and A. Granas, Fixed Point Theory, Springer.

4. Agarwal, R.P., Meehan, M., and O' Regan, D., Fixed Point Theory and Applications, Cambridge Tracts in Mathematics, 141, Cambridge University Press.

Reliability Theory And its Application(PHM-101)

No. of Credits: 4
L T P Total
4 0 0 4

Sessional: 40 Marks
Theory : 60 Marks
Total : 100 Marks
Duration of Exam: 3 Hours

Part-A

Probability generating function: Probability generating function (pgf) of Bernoulli, binomial, Poisson and geometric distributions, Mean and variance of probability distributions using pgf. Mean and variance of probability distributions in terms of Laplace transforms. Stochastic Processes: definition, classification and examples. Markov Chains: definition and examples, transition matrix, order of a Markov chain, Markov chain as graphs.

Part-B

Reliability and Quality. Failure Data Analysis: Failure data, Failure density, Failure rate. Some Important distributions: Exponential, Rayleigh, Weibul, Gamma and Lognormal distributions.

Laplace and Stieltjes transforms and convolutions.

Component Reliability and Hazard Models: Component reliability from test data, Mean time to failure (MTTF), Mean time between failures (MTBF), Time dependent hazard models. Bath-Tub Curve.

Part-C

System Reliability Models: Systems with components in series, Systems with parallel components, k-out-of-m systems, Non-series parallel systems, Systems with mixed mode failures. Standby redundancy: Simple standby system, k-out-of-n standby system.

Part-D

Maintainability and Availability: Maintainability function, Availability function, Reliability and availability analysis of a two-unit parallel system with repair using Markov model, Reliability and availability analysis of single-unit and two- unit cold standby systems with constant failure and repair rates using regenerative point and supplementary variable techniques.

Economics of Reliability Engineering: Manufacture's cost, Customer's cost, Reliability achievement and utility cost models, Depreciation cost models and availability cost model for parallel system.

Books Recommended:

1. E. Balagurusami, Reliability Engineering, Tata McGraw Hill, New Delhi, 1984.
2. L. S. Srinath, Reliability Engineering, Affiliated East West Press, New Delhi, 1991.
3. Elsayed A. Elsayed, Reliability Engineering, Addison Wesley Longman. Inc. Publication
4. A. Birolini, Reliability Engineering: Theory and Practical, Springer-Verlag.
5. Jai Singh Gurjar, Reliability Technology, I.K. International Publishing House Pvt. Ltd
6. Charles E Ebeling, An Introduction to Reliability and Maintainability Engineering, Tata McGraw-Hill Publishing Company Limited, New Delhi,2000

Ph.D Coursework
Operations Research
Code: PHDM-01
No of Credits: 4

		Sessional:	25
L	P	Theory Exam:	75
4	0	Total:	100

UNIT 1

Linear Programming Problem: Computational procedure of simplex method, Alternative optimum solutions: unbounded solutions, non-existing feasible solutions. Big M method, Two phase method, Sensitivity Analysis, Revised Simplex Method, Duality, Dual Simplex Method.

UNIT 2

Inventory Models: Various costs, Deterministic inventory models, Single period inventory model with shortest cost, Stochastic models, Application of inventory models, Economic lot sizes-price breaks.

UNIT 3

Assignment problem, mathematical formulation, solution of assignment problem (Hungarian method), Transportation problem, mathematical formulation. Initial basic feasible solution of transportation problem by North-west corner rule, Lowest-cost Entry method and Vogel's approximation method, Optimal solution of transportation problem.

UNIT 4

Integer linear programming: Integer Linear Programming Problems, Mixed Integer Linear Programming Problems, Cutting Plane Method, Branch and Bound Method.
Queuing theory: Introduction to Queues, Basic Elements of Queuing Models, Queue Disciplines, Role of Exponential and Poisson Distributions, Markovian Process, Erlang Distribution, Distribution of Arrivals, Distribution of Service Times, Definition of Steady and Transient State, Poisson Queues.

Books:

- Hadley, G., "Linear Programming, and Massachusetts", Addison-Wesley
- Taha, H.A, "Operations Research – An Introduction", Macmillian
- Hiller, F.S., G.J. Lieberman, "Introduction to Operations Research", Holden-Day
- Harvey M. Wagner, "Principles of Operations Rsearch with Applications to Managerial Decisions", Prentice Hall of India Pvt. Ltd.
- Swarup K., "Operations Research", S. Chand

PHD – 100A

RESEARCH METHODOLOGY
PhD (Common Subject)

No. of Credits: 4	Sessional: 25 Marks
L | T | P | Total	Theory: 75 Marks
4 | 0 | 0 | 4	Total: 100 Marks
	Duration of Exam: 3 Hours

Course Objectives:

Understand research process in order to plan a research proposal

Learn methods to devise and design a research set-up

Plan and perform data collection methods and its analysis

Conclude research in report writing

Course Outcomes: The research scholar shall be able to

CO1 Plan a research proposal and design the research.

CO2 Collect data through experiments or surveys as per research requirement.

CO3 Understand and apply sampling and sampling distributions.

CO4 Understand and perform quantitative and qualitative data analysis.

CO5 Write research report with proper citations.

Unit 1 **Introduction to Research**: Definition, need and purpose of research, types of research, research process, approaches to research, planning a research proposal, literature review.

Unit 2 **Measurement Scales**: Indexes vs. Scales, Types of Scale, construction of Scale, Bogardus social distance scale, Thurstone Scale, Likert Scale, Semantic Differential Scale, Guttmann Scale.

Unit 3 **Data Collection Methods:** Experiments and Surveys, Experiments: Classical Experiments, Independent & Dependent Variables, Pre Testing & Post Testing, Double Blind Experiment, Subject Selection, Variation on Experiment Design. Survey Research: Topics appropriate for survey research, Guidelines for asking questions, Questionnaire Construction, Strengths & Weakness of Survey Research, Types of Surveys.

Unit 4 **Sampling**: Types of sampling methods: Non Probability Sampling, Probability Sampling, Theory & Logic of Probability Sampling, Sampling Distributions & Estimates of Sampling Error.

Unit 5 **Data Analysis**: Qualitative v/s Quantitative data analysis, Qualitative Data Analysis: Discovering Patterns, Grounded Theory Method, Semiotics, Conversation Analysis, Qualitative Data Processing. Quantitative Data Analysis: Quantification of Data, Univariate Analysis, Bivariate Analysis, Multivariate Analysis, Regression Analysis, Description Analysis. Hypothesis. Multiple Attribute Decision Making.

Unit 6 **Report Writing, Ethical Issues and Outcomes**: Report Preparation, Structure of Report, Report Writing Skills, Citations, Research Papers, Intellectual Property Rights, Plagiarism, Patent, Commercialization, Ethical Issues.

References:

1. Research Methodology by R. Panneerselvam, 2nd Ed. PHI
2. Research Methodology by C.R. Kothari & Gaurav Garg, 3rd Ed. New Age Publishers
3. Research Methodology and Scientific Writing by C. George Thomas, Ane Books
4. The practice of social research by Earl Babbie, 14th Ed. Cengage
5. Multiple Attribute Decision Making, Gwo-Hshiung Tzeng and Jih-Jeng Huang, CRC Press

Ph. D Coursework
Quantum Optics
Code: PHDM-02
No. of Credits: 4

L	P		
4	0	**Sessional:**	**25**
		Theory:	**75**
		Total:	**100**

Course Objectives: To provide elementary knowledge of quantum optics and to learn about the basic concepts and techniques of quantum optics and their applications to physical systems.

Course Outcomes: After the successful completion of the course the research scholar shall be able

CO1: To know about quantization of an electromagnetic field and to comprehend the basic state space representation of the electromagnetic field.

CO2: To apply the displacement operator over the vacuum to formulate coherent state as well as to derive coherent state as an eigenstate of the annihilation operator, to analyse the properties of coherent states.

CO3: To analyse the squeezed state properties and to produce the state by applying the squeezing operator over vacuum.

CO4: To evaluate phase space descriptions of the electromagnetic field like P and Q representations and to demonstrate Wigner's phase space density function for different light states.

Unit I

Quantization of the electromagnetic field, Field quantization, Density of modes, Commutation relations, State space for the electromagnetic field: Fock space and Fock or Number states

Unit II

States of the electromagnetic field I, Coherent states and its properties, coherent states are minimum uncertainty states, coherent states are not orthogonal, coherent states are over complete, Displacement operator and its properties, Photon statistics, Coordinate representation

Unit III

States of the electromagnetic field II, The Squeeze operator, Squeezed states and general properties, Squeezed state is an eigen state of a generalized annihilation operator, calculation of moments with squeezed states, quadrature fluctuations, photon statistics, Multimode squeezed states

Unit IV

Phase space description, Q-representation: anti normal ordering, Normalization, Average of anti normally ordered products, some examples, Density operator in terms of the function Q, Characteristic function, P representation: normal ordering, Normalization, Averages of normally ordered products, Some examples, Wigner distribution: symmetric ordering, Moments

References:

1. P. Meystre and M. Sargent, Elements of Quantum Optics, Springer-Verlag (1990).
2. D. F. Walls and G. J. Milburn, Quantum Optics, Springer-Verlag (1994).
3. G. S. Agarwal, Quantum Optics, Cambridge University Press (2013).
4. Miguel Orszag, Quantum Optics, Springer (2000).
5. M. O. Scully and M. S. Zubairy, Cambridge University Press (2012).
6. R. Loudon, The Quantum Theory of Light, Oxford Science Publications.

Ph. D Coursework
Advanced Mechanics of Solids
Code: PHDM-03
No. of Credits: 4

L	T	P		
4	0	0	**Sessional:**	**25**
			Theory:	**75**
			Total:	**100**

Course Objectives: The objective of this paper is to introduce the concept of strains tensors, stress tensors and basic concepts of elastic body deformation and to make students familiar about the constitutive relations and field equations. Dynamics of elastic bodies and basic problems related to elastic wave propagation are also introduced.

Course Outcomes: After the successful completion of the course the research scholar shall be able

CO1: To know about concept of strains tensors, stress tensors and basic concepts of elastic body deformation.

CO2: To understand the constitutive relations and field equations both mathematically as well as physically.

CO3: To understand two-dimensional propagation of elastic waves**;** waves of dilatation and waves of distortion.

CO4: To derive the frequency equations of Rayleigh-type and Love-type surface waves and also to under reflection/ transmission phenomena from a solid boundary.

Unit I

Tensors: Summation convention, free and dummy suffixes, coordinate transformation, tensors of several orders, Kronecker's delta, Properties of tensors, equality of tensors, scalar multiple of a tensor, sum and difference of tensors, contraction, quotient laws, isotropic tensors, transpose and inverse of a tensor, symmetric and skew-symmetric tensors, invariants of a tensor, deviatoric tensors, eigen space of tensor, comma notation, gradient, divergence, curl tensor notations, Laplacian of a tensor.

Unit II

Stress and Strain: Deformation in elastic bodies, homogeneous strain and its properties, affine transformation, strain tensors, strain-displacement relations, principal strains, strain invariance, stress tensor, components of stress, Equations of equilibrium, generalised Hooke's Law- relation between stress and strain, Elastic constants and their physical significance, Strain energy function and its connection with Hooke's Law.

Unit III

Definition of waves and basic terminology, Harmonic waves, Plane waves, Wave equation in 3-Dimensions, D'Alembert's solution of wave equation, Dispersion of waves and group velocity, Elastic waves, reduction of equation of motion to wave equation, P and S waves, Polarization of S wave, Helmholtz decomposition theorem of a vector, Two-dimensional propagation of elastic waves in isotropic solid, equation of motion in classical theory of elasticity.

Unit IV

Introduction to surface waves, Condition of existence and frequency equation of Rayleigh and Love waves, Particle motion of Rayleigh waves, Frequency equations of Rayleigh waves and Love waves, Snell's law of reflection and refraction, Reflection of plane waves (P/SV and SH-waves) from free surface of an elastic half-space, Reflection and transmission at interface of two different elastic solids, Partition of energy at the interface.

References:

1. Narayan, Shanti. *A text book of Cartesian Tensors (with an introduction to general tensors)*, 3rd edition. New Delhi: S. Chand Publications, 1968.
2. Young, E. C., *Vectors and tensor analysis*, 2nd edition, 1993
3. Sokolnikoff, I. S., *Mathematical theory of elasticity*, 2nd edition, McGraw-Hill, 1982.
4. Chandrasekharaiah, D. S. and Debnath, L. *Continuum Mechanics*, Academic Press Inc., San Diego, CA, 1994.
5. Kolsky, H. *Stress waves in Solids*. Dover Publications, 1963.
6. Ghosh, P. K., *Mathematics of waves and vibrations*. New Delhi: The Macmillan Company of India Limited, 1975.
7. Ewing, W. M., W.S. Jardetzky, and F. Press. *Elastic waves in layered media*. McGraw-Hill Book Co., 1957.

S. No.	Course Name	Course Code	Employability	Entrepreneurship	Skill Development
1.	Operation Research	PHDM-01	✓	✓	-
2.	Quantum Optics	PHDM-02	✓	-	✓
3.	Advanced Mechanics of Solids	PHDM-03	✓	-	✓
4.	Research Methodology	PhD-100A	✓	-	✓

Ph. D Coursework
Approximation Theory
Code: PHDM-04
No. of Credits: 4

L	T	P		
4	0	0	**Sessional:**	**25**
			Theory:	**75**
			Total:	**100**

Course Objectives: The objective of this course is to provide the concepts of best approximation and various tools of approximation.

Course Outcomes: After the successful completion of the course the research scholar shall be able

CO1: To know about different kind of operators like Positive operators, Monotone operators etc.

CO2: To apply the concepts of K-Functional and Moduli of Continuity to find out the rate of convergence of a sequence of linear positive operators.

CO3: To have knowledge of the concept of Best approximation.

CO4: To find the Uniform Convergence of Fourier-Series.

Unit I
Positive linear operators, Monotone operators, Simultaneous approximation, $Lp-$ approximation, Approximation of analytic functions, Big-O and small-o notations and their applications.

Unit II
The Weierstrass's First theorem, The Weierstrass's Second approximation theorem, Bernstein polynomials and their properties, Korovkin theorem, Tchebysheff's theorems, Algebraic and trigonometric polynomials of the best approximation, Lipschitz class, The K-Functional and the Modulus of continuity, Integral modulus of continuity and their properties.

Unit III
Concept of best approximation in a normed linear space, Existence of the best approximation, Uniqueness problem, Convexity-uniform convexity, strict convexity and their relations.

Unit IV
Bernstein's inequality (both first and second), Jackson's theorems and their converse theorems, Approximation by means of Fourier series, Dini-Lipschitz criterion.

References:

1. Cheney, E. W., *Introduction to Approximation Theory*, AMS Chelsea Publishing Co., 1981.
2. Lorentz, G. G., *Bernstein Polynomials*, Chelsea Publishing Co., 1986
3. Natanson, I. P., *Constructive Function Theory, Volume-I*, Fredrick Ungar Publishing Co., 1964.
4. Mhaskar, H. M. and Pai, D. V., *Fundamentals of Approximation Theory*, Narosa Publishing House, 2000.
5. Timan, A. F., *Theory of Approximation of Functions of a Real Variable*, Dover Publication Inc., 1994.
6. Ditzian Z and Totik V., *Moduli of smoothness*, Springer, New York, 1987.